University
of Bremen

– Faculty 3 –

Courses

Summer Semester 2023

M.Sc. Industrial Mathematics & Data Analysis

M.Sc. Mathematics

M.Sc. Mathematik

M.Sc. Technomathematik

April 2023

This brochure summarizes the courses and lectures for the Master's Industrial Mathematics & Data Analysis, Mathematics, Mathematik (German-language), and Technomathematik (German-language) in the summer semester 2023. Further information can be found in the Course Catalog of the University of Bremen. There you will find, among other things, the language, the assignments to the individual modules, and the course code. The latter one can also be used to find a course in Stud.IP.

As you can see in the Course Catalog, all courses are in general assigned with an area of focus or specialization. This can also be found for all courses via *Fields of study* in Stud.IP. For the M.Sc. Industrial Mathematics & Data Analysis, these are Data Analysis and Industrial Mathematics. For the M.Sc. Mathematics and the M.Sc. Mathematik, these are Algebra, Analysis, Numerical Analysis, and Statistics/Stochastics.

At this point we would like to refer to the Offers for International Students as well as to Living on Campus for answers regarding living, housing, financial help, and scholarships.

Contact

Academic Advisory Office - Mathematics

Place to go for questions on study programs, planning, recognition of credits and exam results, study abroad, and examination regulations. Also responsible for the design of this brochure.

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Contents

The lecturers are responsible for the content of the descriptions

Analytic and Discrete Convex Geometry

Course Code: 03-M-SP-15

PD Dr. Eugenia Saorín Gómez

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Course Description

Convex Geometry is, in its origins, the geometry of convex domains in Euclidean space. The theory surrounding it plays a central role in many branches of Mathematics, as Discrete Mathematics, Functional and Harmonic Analysis, Linear Programming, PDEs, and, increasingly, also in the study of algorithms in Computer Science. According to C. Zong (see reference 14 in the literature list), Convex and Discrete Geometry, apart from being one of the most intuitive subjects in Mathematics, *has the characteristic that many of its hardest problems, such as the sphere packing problem or Borsuk's problem, can be explained, along with their conjectured answers, to a layman in a few minutes. However, proofs of the conjectured answers to some of these simply stated problems often have cost the best mathematicians decades or centuries of effort. More surprisingly, some of these commonly believed conjectures, whose truth seemed intuitively certain, were not true. [...]. Furthermore, there are problems in Convex and Discrete Geometry whose answers are so counterintuitive and strange that they can hardly be believed before reading their proofs.*

The main aim of this course is to discuss some of these problems, along with some of their (eventually only partial) answers. For, we will deal with different analytic and discrete aspects of Convex Geometry, keeping the main focus on some of the above-mentioned problems.

As an example of these problems, we briefly describe Borsuk's problem, which asks whether it is true, that any bounded set $X \subset \mathbb{R}^n$ can be partitioned into $n+1$ subsets $X_1, \ldots, X_{n+1}$ such that

$$\operatorname{diam} X_i < \operatorname{diam} X, \quad i = 1, 2, \ldots, n+1.$$

Here $\operatorname{diam} X$ denotes the diameter of X, i.e., as usual,

$$\operatorname{diam} X = \sup_{x,y \in X} ||x-y||.$$

It was widely believed, that the answer to the above question was YES. It is known, that for $n = 3$, the answer is indeed affirmative. Surprisingly, if the dimension n of the space is large enough, the answer happens to be NO.

Course Prerequisites

The course will be self-contained. Although knowledge about the theory of convex sets can be helpful, we only set the usual Linear Algebra and Analysis of the first to third/fourth semesters of a regular Bachelor's degree in Mathematics as prerequisites.
Indeed, the first three to four weeks will be devoted to the basic theory of convex sets in Euclidean space. Depending on the previous knowledge of the participants, we will introduce the essentials of convex sets, some of their (geo)metric properties, and some special families of those, such as polytopes; the space of convex and compact sets in $\mathbb{R}^n$; fundamental geometric magnitudes associated to convex and compact sets, as inradius, circumradius, diameter and (minimal and mean) width; rudiments on convex functions, especially the support function of a convex and compact set; and the fundamentals of (geometric) lattices and packings.

Times and Formalities

The course will consist of three weekly units: two lectures (frontal teaching) and one exercise session.

1. Lectures: Monday 12:00-14:00, in MZH 1100 and Tuesday 12:00-14:00, in MZH 7200.
2. Exercise sessions: Tuesday 14:00-16:00, in MZH 7200.

Depending on the number of students, and upon agreement, there will be a combination of discussion of exercises and short presentations during the exercise sessions.

There will be an oral exam at the end of the term.

Literature

1. S. Artstein-Avidan, A. Giannopoulos and V. D. Milman, Asymtotic Geometric Analysis, Part I, AMS, 2015.

2 I. Bárány, Combinatorial Convexity, AMS, 2021.

2. A. Barvinok, A course in Convexity, AMS, 2002.

3. A. Barvinok, Integer points in polyhedra, EMS, 2008.

4. Y. D. Burago, V. A. Zalgaller, Geometric Inequalities, Springer, 1988.

5. J. Cassels, An Introduction to the Geometry of Numbers, 1971.
6. R. Gardner, Geometric Tomography (2nd Edition), Cambridge, 2006.
7. P. M. Gruber, Convex and Discrete Geometry, Springer, 2007.
8. P. Gruber and C. Lekkerkerker, Geometry of Numbers, 1987.
9. D. Hug and W. Weil, Lectures on Convex Geometry, Springer, 2020.
10. L. Lovász, An Algorithmic Theory of Numbers, Graphs, and Convexity, SIAM, 1986.
11. J. Matousek, Lectures on Discrete Geometry, Springer,2002.
12. R. Schneider, Convex Bodies: The Brunn-Minkowski Theory, (2nd Edition) Cambridge, 2013.
13. G. M. Ziegler, Lectures on Polytopes, Springer, 1995.
14. C. Zong, Strange Phenomena in Convex and Discrete Geometry, Springer, 1996.

Approximation Algorithms

Course Code: 03-IMAT-APX

Prof. Dr. Nicole Megow, Dr. Felix Hommelsheim

Contact: nicole.megow@uni-bremen.de

Description

This course deals with the design and analysis of algorithms for combinatorial optimization problems such as graph problems (matchings, network design, covering problems), scheduling and packing problems. These problems are typically computationally intractable or, in other words, NP-hard. In this course, we design and analyze polynomial-time algorithms for NP-hard combinatorial optimization problems which can provide strong mathematical guarantees on the worst-case performance.

An approximation algorithm is a polynomial time algorithm that always finds a feasible solution whose value is provably close to the optimum solution value. More precisely, the value of the computed solution must be within a factor α of the optimum. Under the common assumption that $P \neq NP$, approximation algorithms are, in some sense, the best algorithms with polynomial running time one can hope to design. Moreover, the performance guarantee α of an algorithm serves as a natural metric to compare the hardness of different problems.

In this course, we will review many classical results in the field of approximation algorithms, highlighting different techniques commonly used for the design of such algorithms. Among others, we will treat the following methods:

- Greedy Algorithms and Local Search
- Rounding and Dynamic Programming
- Deterministic Rounding of Linear Programs (LPs)
- Random Sampling and Randomized Rounding of LPs
- Primal-Dual Methods
- Proving Hardness of Approximation

Format and Examination

The course aims at Master's students, although Bachelor's students in higher semesters are also very welcome. We will teach the course with 4 hours per week (**4 SWS** for **6 CP**), where roughly every other week one class will be an interactive exercise session.
There is the possibility to further extend the content of the course as well as the credits (+ **3 CP**) by participating in a seminar. This participation consists of individually studying a recent research article and presenting the main results to the rest of the class.

Examination: The examination will be by individual oral exam. As admission to the oral exam it is mandatory to present solutions in the exercise session at least twice during the term.

Prerequisites

Having heard an introductory course to discrete algorithms and their mathematical analysis (e.g. "Algorithmische Diskrete Mathematik") is beneficial but it is not a requirement.

Literature

- Williamson and Shmoys: The Design of Approximation Algorithms, Cambridge University Press, 2011.
- Vazirani: Approximation Algorithms, Springer, 2003.

Deep Learning for Inverse Problems and Nonlinear Inverse Problems

Course Code: 03-M-SP-17 & 03-M-SP-18

Dr. Tobias Kluth, Dr. Pascal Fernsel

Contact: tkluth@math.uni-bremen.de, p.fernsel@uni-bremen.de

Course Description

The lectures "*Deep Learning for Inverse Problems*" and "*Nonlinear Inverse Problems*" cover two main topic areas in the field of inverse problems, where each part can be taken individually by the students. Each block takes half of one whole lecture (approximately 7 weeks) and is equivalent to to a course with 2 lecture + 1 exercise hours per week. Taking both lectures corresponds to a course with 4 lecture + 2 exercise hours per week.

Both parts deal with inverse problems. An inverse problem deals with determining the cause (represented by a quantity noted x) for an observed (or desired) effect/measurement (noted y). The nomenclature suggests the existence of a corresponding "direct problem" or "forward problem" which exists in most cases. While it is not always natural to distinguish between a direct and an inverse problem, the concept of ill-posedness helps to characterize the inverse problem. Particularly, for ill-posed inverse problems, the situation becomes delicate if instead of y a noisy version y^δ is solely available, which is likely to be the case in many applications. Small deviations in the measurement result in large deviations in the determined cause quantity.

The **first block** of the lecture called "*Deep Learning for Inverse Problems*" comprises – as the title already reveals – two different fields, namely Inverse Problems and Deep Learning. More precisely, this block reviews well-established solution methods and introduces the students to reconstruction techniques provided by the field of Deep Learning in order to solve applications arising from the field of Inverse Problems. In this block, we mainly focus on linear inverse problems given by a mathematical observation model/forward operator $A : \mathcal{X} \to \mathcal{Y}$ (for suitable spaces $\mathcal{X}, \mathcal{Y}$) linking the cause and the effect. The problem then becomes finding a solution x to the operator equation

$$Ax = y$$

for a given y, resp. noisy y^δ. At the beginning we will briefly review the most important concepts from theory of linear inverse problems and their solution by established regularization methods. Then we will focus on an introduction to deep learning methods, *i.e.* techniques exploiting hierarchical neural networks with a large number of layers for various tasks, for inverse problems. We refer to these methods as "learned methods".

The **second block** of the lecture called "*Nonlinear Inverse Problems*" will have a more theoretical focus and analyzes nonlinear inverse problems by introducing a *nonlinear* forward operator $F : \mathcal{X} \to \mathcal{Y}$ and the corresponding operator equation $F(x) = y$. As before, the corresponding inverse problem consists of finding x based on (a noisy version of) y. Here, we characterize the ill-posedness of the problem, consider Tikhonov regularization techniques for general convex but non-differentiable penalty terms, analyze the general approach with respect to existence, stability, and convergence properties and also consider algorithmic solutions to the problem.

For both the linear and nonlinear case, exciting applications can be found in various fields such as imaging, image processing, parameter identification (e.g., in PDEs) but also mathematical operations such as differentiation fall into this class of problems. We will discuss some of these application examples in both blocks of the lecture.

Course Prerequisites

Knowledge in functional calculus (in particular for the second block) and basic programming skills (in particular for the first block) are required for participation. Basic knowledge of deep/machine learning is necessary. Prior knowledge about linear inverse problems can be beneficial. Most of the programming tools/libraries for common deep learning methods are available in Python. Solutions to programming exercises oftentimes rely on these tools/libraries such that basic knowledge of Python is desirable.

Times and Formalities

Lectures: Tuesdays, 12 - 14 (c.t.), Thursdays, 10 - 12 (c.t.) **Exercise Session**: Tuesdays, 14 - 16 (c.t.)

List of Literature

- H. W. Engl, M. Hanke and A. Neubauer. *Regularization of Inverse Problems.* Kluwer Academic Publishers Group, 1996.
- Bangti Jin and Peter Maass. *Sparsity Regularization for Parameter Identification Problems.* In: *Inverse Problems* 28.12 (2012), p. 123001.

Further references will be provided during the lecture.

Digital Optimal Control and Feedback Control

Course Code: 03-M-SP-20

Prof. Dr. Christof Büskens

Contact: bueskens@uni-bremen.de

Course Description

Our interactions with the environment can be divided into two categories: Observation and influence. Through observation of nature we gain knowledge about the nature of the universe which enables us to describe dynamical systems of real processes (such as for spacecraft, robots, chemical reactions or economic processes) mathematically. Influencing these systems should not take place in an aimless and uncontrolled manner. But rather the optimisation of a performance measure, subject to certain constraints, is in the foreground. Euler (1707-1783) already recognised:

Since the form of the whole universe is most perfect, designed by the wisest of all creators, nothing happens in the world, without that somehow a maximum or minimum rule shows up.

A special role is played here by (optimal) control, feedback control of *dynamical systems*, where the influence is exerted along a free variable (usually the time). In this lecture, the differences between control and feedback control will be worked out and it will be shown how far mathematical theory can be taken before numerical iterative methods should take over.

Ergodic Theory

Course Code: 03-M-SP-13

Prof. Dr. Anke Pohl

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Course Description

Does the trajectory of a billiard ball come arbitrarily close to any point on the billiard table or are certain regions preferred? How well can you approximate real numbers with the values of quadratic forms? How well and how quickly can irrational numbers be approximated with fractions? Do chocolate drops in the cake dough collect in one place when they are stirred for a long time? How can dynamical systems be used to detect tax fraud?

Such questions can be answered with the help of ergodic theory, a branch of mathematics that studies the long-term behavior of dynamical systems in terms of probabilistic and measure-theoretic aspects. The origin of ergodic theory lies in statistical physics (which will not play a role in the lecture, in particular no knowledge of physics is required); nowadays the methods and results of ergodic theory are applied in many areas of mathematics.

We will cover the following topics, among others:

- Recurrence
- Ergodicity and ergodic theorems
- Equidistribution
- Entropy theory
- Applications to selected mathematical problems
- Further selected topics, e.g., dynamics of continued fractions or mixing

Course Prerequisites

Solid knowledge of the mathematics courses from a B. Sc.in mathematics, in particular:

- comprehensive knowledge of Real and Complex Analysis as well as Linear Algebra

- familiarity with Measure Theory
- basic elements from Point-Set Topology (metric spaces are sufficient)
- fundamental theorems of Functional Analysis (Theorems of Hahn–Banach, Arzela–Ascoli, Choquet, etc.)
- familiarity with Fourier Theory

You can find the necessary background material in many classical textbooks. Good sources are also the appropriate sections in

Einsiedler, Ward: *Ergodic theory. With a view towards number theory.*
Springer Graduate Texts in Mathematics 259 (e-book available in SuUB).

If you wish more reference, then please contact me. During the lecture course, we can do brief surveys only, no major recapitulations.

Times and Formalities

Type of lecture: 4 SWS class room lecture, 2 SWS tutorials (for precise schedule see Stud.IP)

Classification: Elective module in M. Sc.Mathematics (Specialization Analysis)

Exam: oral

Studienleistung: successful completion of homework and active participation at tutorials (details in Stud.IP)

List of Literature

- Lecture notes will be provided (in English)
- Einsiedler, Ward: *Ergodic theory. With a view towards number theory.*
  Springer Graduate Texts in Mathematics 259
  (e-book available in SuUB)
- further recommendations during the course of the lecture

Evolution Equations and Form Methods

Course Code: 03-M-SP-24

Dr. Hendrik Vogt

Contact: hendrik.vogt@uni-bremen.de

Course Description

This course deals with the functional analytic treatment of evolution equations. Roughly speaking we will investigate initial value problems of the type

$$u'(t) = Au(t), \quad u(0) = u_0.$$

At the first sight this looks like an ordinary differential equation; however, A will not simply be a matrix but an unbounded linear operator in a Banach space X. A prototypical example is the Laplace operator $A = \Delta$ in $L_2(\mathbb{R}^n)$ (with a suitable domain of definition); then $u' = Au$ becomes the heat equation on $\mathbb{R}^n$.

The first task will be to work out for which operators A the above initial value problem is *well-posed*, i.e., under what conditions one obtains a unique solution that depends continuously on the initial value u_0. This question will be answered in the general framework of C_0-semigroups.

The main topic of the course is the construction of specific operators A for which one has well-posedness; this will be achieved by form methods. Applications include elliptic operators with various boundary conditions (Dirichlet, Neumann, Robin) and the Dirichlet-to-Neumann operator. (The latter operator maps a given stationary temperature distribution on the surface of a body to the corresponding heat flux through the surface.)

Course Prerequisites

This course is intended for Master students with a good knowledge of basic Functional Analysis. Some knowledge of Partial Differential Equations is helpful for following the lecture, but not strictly necessary. In addition, results from Complex Analysis up to Cauchy's integral formula are required.

Times and Formalities

This is a classical 4+2 course with two 2-hours lectures and one 2-hour exercise session per week. There will be a homework sheet each week; students are supposed to present their solutions during the exercise sessions. The examination will be an oral exam in the semester break.

List of Literature

- Lecture Notes of the 18. Internet Seminar, see https://www.mat.tuhh.de/home/pbaasch/isem18/Phase_1_The_lectures.html

Hands-On Tutorial on Optimization

Course Code: 03-IBFW-HTO

Prof. Dr. Nicole Megow

Contact: nicole.megow@uni-bremen.de

Description

A large number of problems arising in practical scenarios like communication, transportation, planning, logistics etc. can be formulated as discrete linear optimization problems. This course briefly introduces the theory of such problems. We develop a toolkit to model real-world problems as (discrete) linear programs. We also explore several ways to find integer solutions such as cutting planes, branch & bound, and column generation.
Throughout the course, we learn these skills by modeling and solving, for example, scheduling, packing, matching, routing, and network design problems. We focus on translating practical examples into mixed-integer linear programs. We learn how to use solvers (such as CPLEX and Gurobi) and tailor the solution process to certain properties of the problem. We also give some theory background on linear programming, polyhedral theory and solutions methods.
This course aims at students with an interest in practical applications. Some background on algorithmic discrete math and/or discrete optimization is advantageous but not necessary. As this course does not require any prerequisites except some basic programming skills, it is announced as a Bachelor's course (**6 CP**). Interested Master students are very welcome and will not be bored.

Format and Examination

This course consists of two phases:

- **One week** Mon-Fri (fullday) of lectures and practical labs.
- An individual project period: One project has to be modeled, implemented, and solved individually or in a group of at most two students. The topic will be either developed with or provided by the lecturers.

We expect active participation during the week. The project results including the implementation and a written documentation have to be presented in the beginning of the winter semester and will be graded.

High-Performance Visualization

Interactive Visualization of Huge Scientific Datasets

Course Code: 03-M-SP-12

Prof. Dr. Andreas Gerndt

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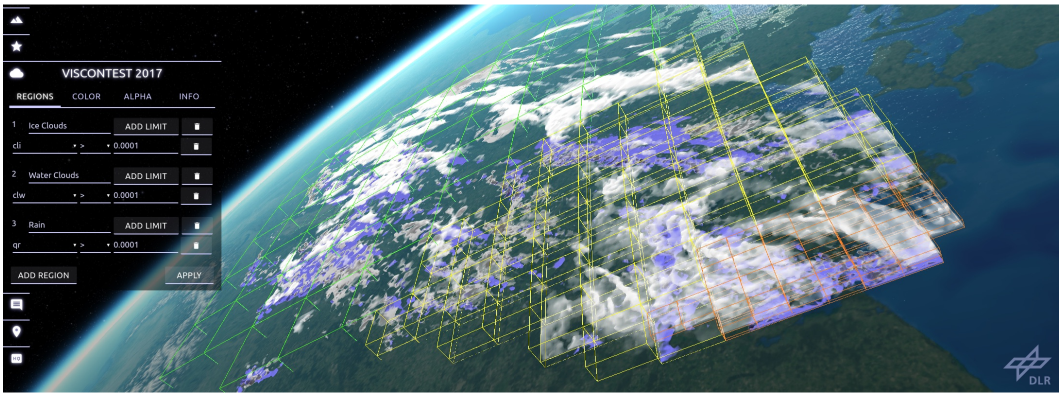
Homepage: https://www.uni-bremen.de/ag-high-performance-visualization



1 Ice Clouds
cli > 0.0001

2 Water Clouds
clw > 0.0001

3 Rain
qr > 0.0001

Course Description

In this lecture, the mathematical basics of scientific visualization are taught. It aims at methods for parallel post-processing of very large-scale scientific datasets. Such data occurs in plenty of scientific applications. It is created by simulations on high-performance supercomputers (e.g. to support climate research or analysis of flow fields at airfoils). It can also be the outcome of measurements as it occurs in Earth observation missions. To get any insight into the scientific results, first of all, a huge amount of raw data has to be processed to extract meaningful features. Those features can eventually be explored in interactive working environments. To enable real-time exploration at the end of the processing pipeline, again highly parallel and efficient methods are required. They have to be optimized for the execution on distributed computing clusters and high-end graphics cards. This lecture addresses foundational approaches of feature extraction, data processing, and efficient 3D visualization. Application examples are demonstrated with the Open Source software ParaView.

Course Prerequisites

Students from Mathematics, Computer Science, and other relevant application domains (like Geo-science or Aerodynamics) can participate at the lecture. Background knowledge in Computer Graphics or High-Performance Computing is useful but not required. Programming skills e.g. in Python or C++ are also useful.

Times, Formalities, and Examination

In weekly lectures, several topics are presented: Computer Graphics Primer, High-Performance Computing Primer, Visualization Pipeline, Data Representation and Reconstruction, Scalar Visualization, Color Mapping, Scalar Topology Extraction, Vector Field Processing, Particle Integration, Vector Field Topology, Tensor Field Visualization, Direct Volume Rendering, Parallel and Distributed Post-processing, Multi-Resolution and Data Streaming, In-situ Co-processing, Terrain Rendering, Atmosphere Visualization, Flow Visualization, Vortex Extraction, Multivariate Data Queries.
The lecture is given in English. The slides are in English as well and can be used as references. Application exercises and example datasets are provided to repeat the presented topics. Programming exercises can also be carried out as homework. In the last lecture, those programming results can be presented which will become then part of the evaluation. Consultation hours can be agreed on personal need. The lecture eventually ends with an individual oral exam.

List of Literature

- A. C. Telea, "Data Visualization – Principles and Practice", 2. Edition, CRC Press, 2015

- E. W. Bethel, H. Childs, C. Hansen, "High Performance Visualization", CRC Press, 2013

- W. Schroeder, K. Martin, B. Lorensen, "The Visualization Toolkit", 4. Edition, Kitware, 2006

- C. Hansen, C. Johnson, "The Visualization Handbook", Elsevier Academic Press, 2005

Linear Regression Analysis with R

Course Code: 03-M-SP-21

Dr. Maryam Movahedifar

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Course Description

The regression methods are widely used in different fields of study like social sciences, economics, biology, and engineering. Regression analysis is a statistical process that enables the prediction of relationships between variables. The predictions are based on the casual effect of one variable upon another. Regression techniques for modeling and analyzing are employed on a large set of data in order to reveal the hidden relationship among the variables.

This course will give you a run-down explaining what regression analysis is, explaining to you the process from scratch. The first few sessions will be given an understanding of what the different types of learning are – supervised and unsupervised, and how these learnings differ from each other. We then move to cover the supervised learning in detail covering the various aspects of regression analysis. The outline of lectures is arranged in a way that gives a feel of all the steps covered in a data science process – loading the training dataset, handling missing values, transformations, and feature engineering, model building, assessing the model fitting and performance, and finally making predictions on unseen datasets. Each lecture will be started by explaining the theoretical concepts and then we move to the practical examples to support the understanding. The practical examples are illustrated using R code including the different packages in R such as R Stats, Caret, and so on. Each lecture is a mix of theory and practical examples.

By the end of this course, you will know all the concepts and pain points related to linear regression analysis, and you will be able to implement your learning in your projects. Also, this course can be extended for the next semester as Non-Linear Regression Analysis with R.

Course Prerequisites

- No formal requirements, but knowledge of proper backgrounds of statistical concepts and R software is strongly recommended.

- Sprache: English.

Times and Formalities

This course will be presented as lectures plus exercises = 4,5 CP as an in-person class.
Regular participation in the course, working on classroom tasks, working on 50% of the homework in a meaningful way and a final exam, will determine the final score.

Literature

- Regression Analysis with R: Design and develop statistical nodes to identify unique relationships within data at scale, Giuseppe Ciaburro, 2018.
- Statistical Regression Modeling with R, Ding-Geng (Din) Chen, Jenny K. Chen, 2021.

Mathematical Foundations of Machine Learning

Course Code: 03-M-SP-16

Dr. Matthias Beckmann, Prof. Dr. Peter Maaß

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Course Description

Machine learning is nowadays an integral component of intelligent systems for analysing data in natural and engineering sciences. Machine learning algorithms are used in a wide variety of applications, such as in medicine, bioinformatics, natural language processing, speech recognition and computer vision, where it is difficult or infeasible to develop conventional algorithms to perform the needed tasks.

For example, it allows to predict the binding behaviour of complex molecules in drug design or to process a huge amount of sensor data in real time to pave the way for autonomous driving.

This course gives an introduction into the mathematical foundations of machine learning and focuses on classical theory as well as the implementation of basic methods. In particular, the following topics will be covered:

- Elements of statistical learning theory
  - PAC learning,
  - VC theory,
- Methods of supervised learning
  - classification,
  - regression,
- Kernel methods,
- Generative models
  - MAP estimation,
  - mixture models,
- Methods of unsupervised learning
  - manifold learning,
  - cluster analysis.

Among others, we will make use of tools from probability theory, statistics, functional analysis, (convex) optimization and approximation theory.

Course Prerequisites

Basics from B.Sc. courses in Mathematics (calculus, linear algebra, numerical analysis, probability theory) and basic programming skills. In particular, participants are assumed to be familiar with the following standard concepts:

- eigendecomposition, singular value decomposition, multivariate calculus, Landau symbols,
- probability space, measurability, random variable, probability distribution, probability density function, expected value, joint probability, marginal probability, conditional probability, independence, Boole's inequality, Markov's inequality, Chebychev's inequality, Bayes' theorem,
- normed space, Banach space, bilinear form, inner product, Hilbert space, Cauchy-Schwarz inequality, orthogonal decomposition, linear operator, operator norm, orthogonal projection, linear functional, dual space.

Times and Formalities

The course, comprising 4+2 hours per week, is split into a lecture series (4h each week), presumably

Wed. 10-12 in MZH 1470 & Fr. 12-14 in MZH 2340,

and accompanying exercise classes (2h each week), presumably

Fr. 14-16 in MZH 2340.

It is necessary to solve the provided exercise sheets and actively participate in the exercise classes. Exercises will be assigned every week and the students are requested to present their solutions during the exercise classes.

List of Literature

- M. Mohri, A. Rostamizadeh, A. Talwalkar, *Foundations of Machine Learning*, MIT Press, 2. Edition, 2018.
- K. Murphy, *Probabilistic Machine Learning: An introduction*, MIT Press, 2021.
- S. Shalev-Shwartz, S. Ben-David, *Understanding Machine Learning: From Theory to Algorithms*, Cambridge University Press, 2014.

Mathematics of Quantum Computing

Course Code: 03-M-SP-19

Dr. Matthias Knauer

Contact: knauer@uni-bremen.de

Course Description

The need to process ever larger amounts of data means that the development of ever smaller circuits and ever more compact physical memory is constantly being promoted. At the latest at the sub-atomic level, the effects of quantum mechanics must be taken into account in order to be able to describe calculation rules and data storage. This is what the young discipline of „quantum computing“ deals with.
In addition to discussing physical principles, we need mathematical foundations from many different areas: analysis, linear algebra, functional analysis, group and number theory, and stochastics.

The aim of this course is to create an understanding of which algorithms are suitable for implementation on quantum computers and which problems can be solved with them.
To do this, we study the basic terms of quantum mechanics and introduce the concept of quantum bits (qubits) as the smallest unit to store data (instead of classical bits). Here we encounter the curious property of entanglement, and illustrate this with the Einstein-Podolski-Rosen paradox and Bell's inequality. We need quantum gates to represent arithmetic operations and can use them to build quantum circuits.

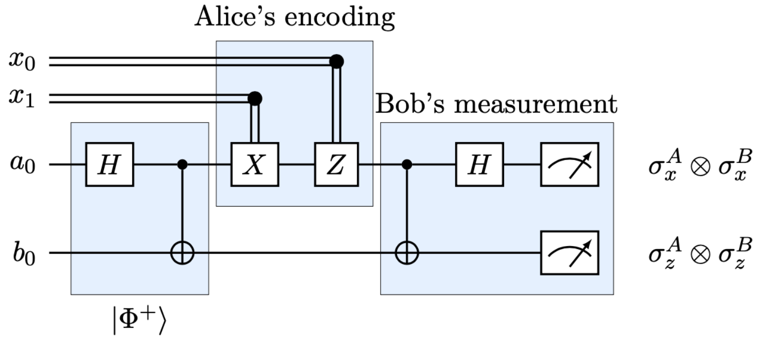

Figure 1: Alice sends two classical bits x_0 and x_1 to Bob using one qubit a_0.

Using various algorithms (e.g. Deutsch-Jozsa algorithm, Shor's algorithm) and tasks (detection of attacks on communication) we discuss the differences between classical computing and quantum computing.
We close the lecture with contributions on error correction (which plays an important role to actually build quantum computers) and on adiabatic quantum algorithms (which use quantum mechanical states directly and are no longer based on circuit concepts).
In addition to the mathematical part, we will also build quantum circuits in the Python-based programming language Qiskit and simulate resp. execute them on the IBM Quantum Lab computers.

Course Prerequisites

- Functional analysis

Times and Formalities

The course consists of a 4-hour lecture and a 2-hour exercise and is held in English.

- Lecture on Monday 12-14 and on Tuesday 8-10
- Exercise on Wednesday 12-14

The lecture notes with the current material will be provided to the students each week.
The students' solutions of the weekly homework has to be submitted as PDF. The solutions to the tasks are discussed in the exercise sessions.

Exam

- Requirement #1: 50% of the points of the exercise sheets
- Requirement #2: 2× successful presentation of own solution in exercise session
- Oral exam. Several selection dates by arrangement.

List of Literature

- Wolfgang Scherer: Mathematics of Quantum Computing. Springer, 2019.

- Matthias Homeister: Quantum Computing verstehen. Springer, 2018.
- Abraham Asfaw et al.: Learn Quantum Computation using Qiskit, 2020.

Optimal Control in Function Spaces

Course Code: 03-M-SP-11

Prof. Dr. Andreas Rademacher

Contact: arademac@uni-bremen.de

Course Description

At the beginning of the course we will first deal with the basics of functional analysis, in particular with differentiation in Banach spaces. Then we discuss set-valued mappings in detail. This lays all the foundations for the main part of the lecture, which focuses on necessary and sufficient optimality conditions for constraint minimization problems. Furthermore, we discuss the application of the obtained results in the field of optimal control. The underlying models, which we consider in this lecture, are based on ordinary differential equations.

Course Prerequisites

The essential basics from calculus and functional analysis should be known. This includes in particular knowledge of Lebesgue and Sobolev spaces.

Times and Formalities

- Lectures: Mondays 10:15 and Thursdays 8:15 in MZH 2340
- Exercise sessions: Mondays 8.15 in MZH 2340
- Weekly homework
- Oral exam

List of Literature

- W. Schirotzek: Nonsmooth Analysis. Springer, Berlin, 2007.
- F. Tröltzsch. Optimal Control of Partial Differential Equations. AMS, 2010.

Regression Models (Statistics II)

Course Code: 03-M-SP-22

Prof. Dr. Werner Brannath

Contact: brannath@uni-bremen.de

Course Description

The lecture and exercises will provide a sound introduction to the realm of statistical modelling with a focus on linear and generalized linear models. We will formally introduce these regression models, discuss the related statistical inference and hypothesis testing problems as well as the mathematical and statistical properties of their solutions. We will also give an introduction to basic variable selection and model building methods and discuss them in light of the fundamental trade-off between bias and variance. We will also discuss the rational, application and limitations of the models and introduced statistical methods. The exercises course will provide the opportunity to train theoretical as well as practical tasks, the latter with the open source software R.

Course Prerequisites

Subject-related prerequisites are basic knowledge of stochastics and statistics, as taught e.g. in the lectures "Stochastics" and "Basics of Mathematical Statistics" at the University of Bremen. Registration at Stud.IP is a necessary prerequisite for participation.

Times and Formalities

The lecture will take place at every Tuesday and Thursday, from 8:15 to 9:45. The exercise course will be on Friday, 8:15-9:45. The lecture halls and seminar rooms are announced in StudIP.

List of Literature

Literature will be announced in the first lecture and via StudIP.

Sequential and Adaptive Designs

Course Code: 03-M-SP-23

Prof. Dr. Werner Brannath

Contact: brannath@uni-bremen.de

Course Description

This course provides an introduction to statistical designs for sequential and adaptive studies and the corresponding statistical methodology. Sequential designs are for empirical (mainly clinical) studies where the data are analyzed not only once at the end of the trial, but also at one or several interim analysis. Due to the repeated testing and thereby increased type I error rates, specific test and estimation procedures are required. The course will cover the basic concepts and different design options for group sequential studies and will carefully discuss their properties. We will also discuss the issue of sample size estimation and mid-trial sample size reassessment. The latter will bring us to the concept and methodology of adaptive designs, which permit for data driven design adaptations in the course of a study. In order to control the type I error rate new statistical concepts are required which follow the so called conditional invariance principle. The course will given an introduction to this principle and the corresponding statistical methods (e.g. combination tests).

Course Prerequisites

Subject-related prerequisites are basic knowledge of statistics, as taught e.g. in the lectures “Basics of Mathematical Statistics” or “Statistical Modelling I” at the University of Bremen. Registration at Stud.IP is a necessary prerequisite for participation.

Times and Formalities

The course will be on Wednesday from 12:15 to 13:45 (lecture) and 14:00 to 14:45 (exercises). It will take place in the seminar room of the KKSB (Linzer Str. 4, ground floor).

List of Literature

- Wassmer, G., Brannath, W. (2016). *Group Sequential and Confirmatory Adaptive Designs in Clinical Trials*. Springer, Switzerland.
- Jennison, C., Turnbull, B. W. (2000). *Group Sequential Methods with Applications to Clinical Trials*. Chapman & Hall/CRC: Boca Raton

Scientific Programming and Advanced Numerical Methods

an introduction with case studies

Course Code: 03-M-SP-14

Prof. Dr. Stephan Frickenhaus, Prof. Dr. Alfred Schmidt

Contact: stephan.frickenhaus@awi.de, alfred.schmidt@uni-bremen.de

Course Description

Research software development deserves a systematic approach to keep up with the demand for reproducible science and reuse of codes as a citeable scientific output. In the context of Open Science, sustainable research software is more and more estimated as vital component of research infrastructures. This course provides an introduction to the practice of scientific programming to a broader audience. The basis are advanced numerical methods and real world research codes that will be explored and executed on local programming environments - either on students laptops or on central compute nodes at University Bremen.

Principles of code-management and code publication will be actively explored in small practical projects, open for students interests in bringing their own software projects. Some main "code use cases" are provided, based on Fortran and C programming language, and further projects are offered for R and C++ as well. Special emphasis is laid on performance optimization and two standard approaches of parallelization, i.e. loop parallelization and domain decomposition.

Advanced numerical methods like domain decomposition, efficient solvers for large systems of equations, preconditioning, etc. will be presented and discussed.

The course is useful for math students and interested participants from other fields (e.g. industrial math, numerics of PDEs, modeling seminar, material sciences - ProMat).

Course Prerequisites

Your own laptop computer is needed and it will make your project work at home easier. We'd install a working programming environment on the basis of WindowsSubsystemLinux (WSL) or Cygwin, or enable access to the

ZeTeM cluster via VPN. We use open Gnu-Software of programming language compilers.

Times and Formalities

4 SWS on a weekly basis, practicals are in blocks; the course offers 9 EC.
You will work with your laptops and access the computers at ZeTeM via VPN.
For successful participation we expect a project report or a “code talk” with demonstration how code was adopted, tested and extended, including documentation, possibly towards publication.

Advanced Topics in Inverse Problems

Course Code: 03-M-AC-13

Dr. Tobias Kluth

Contact: tkluth@math.uni-bremen.de

Course Description

An inverse problem deals with determining the cause (represented by a quantity noted x) for an observed (or desired) effect/measurement (noted y). The nomenclature suggests the existence of a corresponding "direct problem" or "forward problem" which exists in most cases. While it is not always natural to distinguish between a direct and an inverse problem, the concept of ill-posedness helps to characterize the inverse problem. Particularly, for ill-posed inverse problems, the situation becomes delicate if instead of y a noisy version y^δ is solely available, which is likely to be the case in many applications. Small deviations in the measurement result in large deviations in the determined cause quantity.
In this seminar we will cover advanced topics in regularization theory for inverse problems as well as modern regualrization techniques including learning based methods. Topics will be introduced and assigned during the first week of the lecture period.

Course Prerequisites

Knowledge in functional calculus is required for participation. Prior knowledge on inverse problems is also necessary or might be gained simultaneuosly in the inverse problems lectures which are also offered during summer term. Basic knowledge of deep/machine learning can be beneficial.

Times and Formalities

Seminar: Tuesdays, 16 - 18 (c.t.)

Geometry

Course Code: 03-M-AC-11

Dr. Tim Haga

Contact: timhaga@uni-bremen.de

Seminar Description

Are you fascinated by geometric shapes and their properties? Are you an Bachelor's or Masters's student in mathematics looking to deepen your understanding of polytopes? Then this seminar is for you!

In this seminar, we will delve into the fascinating world of polytopes, which are geometric objects that generalize the idea of a convex polygon or a convex polyhedron. Polytopes have a wide range of applications in various fields, such as computer graphics, physics, and optimization.

We will cover a variety of topics related to polytopes, including:

- Basics of polytopes: We will begin by discussing the basic properties of polytopes. We will also explore the different types of polytopes, including simplices, cubes, and cross-polytopes.
- Combinatorial properties: We will delve into the combinatorial properties of polytopes, including the Euler characteristic, face numbers, and the Dehn-Sommerville equations.
- Algebraic properties: We will examine the algebraic properties of polytopes, including their symmetry groups and the representation of polytopes by linear inequalities.
- Applications: We will explore the different applications of polytopes in other areas of mathematics and science, such as linear programming, coding theory, and computer graphics.
- Computational methods: We will learn about computational methods for dealing with polytopes, including convex hull algorithms, triangulation algorithms and algorithms for counting the number of faces of a polytope.

Throughout the seminar, we will use examples and case studies to illustrate the concepts and theories discussed. The seminar will also touch on the recent developments in the study of polytopes, including new results and open problems.

But this seminar is not just about listening and taking notes, we want you to actively participate in the class. Each student is expected to give a talk on a specific topic related to polytopes, allowing you to not only deepen your own understanding but also share your knowledge with your peers. This seminar is also a great opportunity for students to connect with other students with similar interests and to learn from each other.
Join us on a journey of discovery and exploration as we uncover the hidden beauty and complexity of polytopes.
Sign up now and be ready to be amazed!

Seminar Prerequisites

In general, some basic knowledge in the following areas would be beneficial:

- Linear algebra: Familiarity with concepts such as vectors, matrices, and linear transformations is necessary in understanding the algebraic properties of polytopes.
- Calculus: Knowledge of basic calculus concepts such as limits, derivatives and integrals is useful for understanding some of the concepts that will be covered in the seminar.
- Geometry: A basic understanding of Euclidean geometry would be beneficial to understand the concepts of polytopes.
- Basic concepts of combinatorics: Familiarity with counting principles and graph theory would be beneficial in understanding the combinatorial properties of polytopes.

It should be sufficient if you have completed a course in Linear Algebra, Convex Analysis, Combinatorics, or Computational geometry before taking this seminar.
If you have any questions, please do not hesitate to contact the Seminar organizer, Tim Haga.

Times and Formalities

The first seminar session will take place on Tuesday, April 11th, 10 a.m. in MZH 7200. There, we will distribute the topics and set the other seminar dates.
This seminar is suited as well for Master's and Bachelor's students. The seminar will be held in English.

Literature

- *Convex Polytopes* by Branko Grünbaum, which is considered a classic in the field and provides a comprehensive introduction to the theory of convex polytopes.
- *Lectures on Polytopes* by Günter M. Ziegler, which provides an introduction to the theory of polytopes and its applications in mathematics and computer science.
- *Combinatorial Convexity and Algebraic Geometry* by Günter Ewald, which covers the connection between convexity and algebraic geometry, with a focus on the combinatorial aspects of polytopes.
- *Geometric Regular Polytopes* by Peter McMullen, which focuses specifically on regular polytopes, which are a special class of polytopes that have a high degree of symmetry.

Multiple Testing Procedures

Course Code: 03-M-AC-12

Dr. Anna Vesely

Contact: vesely@uni-bremen.de

Course Description

In real data analysis, researchers are often interested in using the same data set to make inference on multiple hypotheses. For instance, they may want to identify brain regions that are activated by a stimulus in brain imaging data, or biological pathways that are differentially expressed in genomics data. However, testing multiple hypotheses simultaneously is a non-trivial extension of the individual case.

Indeed, any hypothesis test carries the risk of making a type I error, i.e., falsely rejecting a true hypothesis; this leads to a 'false discovery' and so a potentially misleading scientific result. Standard methods for testing an individual hypothesis allow to bound the probability of making such an error by an 'acceptable' risk, usually set at 0.05. When performing multiple tests, however, each one has a probability of producing a type I error. As a result, the risk of having at least one error among the findings may become unmanageable.

In this seminar we will introduce the problem of multiple testing and explore different techniques that have been proposed to control the flood of type I errors that arise in this context. We will see that different procedures aim at controlling different generalizations of the type I error. First, we will focus on procedures that control either the familywise error rate (FWER) or the false discovery rate (FDR), a class that includes many of the most popular multiple testing methods. Subsequently, we will introduce recent proposals that estimate the false discovery proportion (FDP) or provide confidence intervals for it.

The seminar is held in English.

Course Prerequisites

No formal requirements, but knowledge of proper backgrounds of statistical concepts and methods is strongly recommended.

Times and Formalities

The seminar takes places once per week in two-hour sessions (90 minutes). In the first session the subject will be introduced and each student will receive one/two topics. Each of the subsequent sessions will consist of a student's talk on their topic (approximately 70-75 minutes), followed by a discussion. The seminar gives 4.5 ECTS credit points (CPs) (6 CPs in the old Master's program).
According to the final number of participants, CPs will be gained through one of the following modalities:

- a talk and a written draft on a topic
- two talks on two different topics

The final grade will depend on the student's performance on the above, but also on the quality of their contribution to the discussions of other students' talks.

List of Literature

- Dickhaus, T. (2014), *Simultaneous Statistical Inference With Applications in the Life Sciences.* https://doi.org/10.1007/978-3-642-45182-9.
- Goeman, J. J. and Solari, A. (2014), Multiple hypothesis testing in genomics. *Statist. Med.*, 33:1946-1978. https://doi.org/10.1002/sim.6082.
- Other sources will be made available during the course of the seminar.

Modeling Project (Part 1)

Course Code: 03-M-MP-1

Prof. Dr. Andreas Rademacher

Contact: arademac@uni-bremen.de

Course Description

Over the course of two semesters, the participants of the modeling project work in teams on a project in which they are supposed to use the mathematical knowledge they have already acquired in applications outside of mathematics. The project partners can be industrial companies or research institutes. The range of topics is determined by the offers of the project partners. This year we are looking forward to cooperation with these partners (in alphabetical order):

- Alfred-Wegener-Institute (AWI, Bremerhaven)
- Bosch
- Bruker Daltonics (Bremen)
- German Aerospace Center (DLR): Institute for the Protection of Maritime Infrastructures (Bremerhaven)
- KUKA Assembly & Test (Bremen)
- Sikora (Bremen)
- to be continued

Course Prerequisites

The modeling project is aimed at students in the Master's programme in Industrial Mathematics and Data Analysis. In limited exceptions, students in the Master's programme in Mathematics can also participate.

Times and Formalities

- Regular meetings
- Presentations by the participants on their topics and the current status of their work

The assessment will take place at the end of the modeling project (Part 2) in February 2024 on the basis of these submissions:

- Internal mathematical presentation and public user-oriented presentation
- Written elaboration (approx. 30 pages)
- Poster or comparable format, e.g. video, interactive software, demonstrator

Reading Course Algebra

Course Code: 03-M-RC-ALG

Prof. Dr. Dmitry Feichtner-Kozlov

Contact: dfk@math.uni-bremen.de

Course Description

Independent study of selected topics of the mathematical area algebra using monographs and research articles.
The goal of the course is to familiarize students with selected topics in the area specialization algebra via books, articles and other specialized literature. This will take place under the guidance of an independent teaching assistant in algebra and related areas (as well as geometry or topology).

Times and Formalities

Beside your self-study, there will be regular meetings to discuss the in an informal or formal manner. Also written reports are mandatory and the course may include an introduction to in-depth fundamentals. This should ideally be used to familiarize the student with topics related to a Master's thesis.
You choose your supervisor; the coordinator, Prof. Dmitry Feichtner-Kozlov will be happy to advise you. You should first discuss the topic of the reading course with the supervisor. This person should then contact the coordinator and agree on the content.
Successful participation will be certified upon request at the end of the Reading Course by the coordinator in consultation with the supervisor.
Please also refer to the module description in the module handbook. Supervision by a university lecturer, or a research assistant from the ALTA Institute is professionally obvious.
All further achievements (typically a written paper and a longer presentation) will be agreed upon as part of the supervision.

Reading Course Analysis

Course Code: 03-M-RC-ANA

Prof. Dr. Anke Pohl

Contact: apohl@uni-bremen.de

Course Description

In the Reading Course Analysis we study a topic in the wider realm of analysis in more depth. This Reading Course can serve as a basis for master theses. In contrast to the past semesters, we will try the following experiment for the summer semester to find a nice topic. Well advance of the lecture period, we will collect suggestions from you as well as from me. In the first week of the lecture period, we will decide on the topic. More details on this decision process can be found in Stud.IP.

Therefore, if you are interested in taking a RC Analysis, then please register in Stud.IP as soon as possible and contribute to the discussion of potential topics. You can deregister at any time.

Course Prerequisites

This Reading Course is a master's level course. Solid knowledge of Analysis 1–3 and Linear Algebra 1–2 is essential. Depending on the chosen topic, familiarity with Complex Analysis, Topology (point-set), Functional Analysis or Measure Theory might be needed.

Format and Form of Examination

The Reading Course is a 9 CP course, i.e.it is of similar scope as a standard lecture course. It belongs to the Specialization Analysis. Organizationally, this Reading Course consists of the following parts:

- You will work on certain sections of the reading material at home (alone, or in informal groups) within a set period of time. We will determine the relevant parts of the reading material in due time.
- We meet regularly all together and discuss the contents of the reading material, questions, etc. From time to time you present short sections at the meeting. These presentations are much less extensive than seminar presentations; they serve to support the discussions. *The schedule for these meetings will be agreed on with all participants.*

The exam consists of active participation and a written paper of about 20 pages covering the content of a smaller section. The examination is ungraded. The language of this reading course (German/English) depends on the participants.

Reading Course Numerical Analysis

Course Code: 03-M-RC-NUM

Lecturers of the ZeTeM

Contact: bueskens@uni-bremen.de, knauer@uni-bremen.de

Course Description

Students study special topics of numerical analysis in this reading course. The aim is a self-study of selected topics on the basis of textbooks, scientific articles or other monographs. The course may also include an introduction into other associated topics or to special software (e.g. Alberta, WORHP). In addition, aspects of scientific work will be discussed, e.g. obtaining relevant literature, correct citation, or structure of a scientific article. All this is done under the supervision of a lecturer from the ZeTeM. In addition to the self-study, there will be regular meetings with the supervisor to discuss the topics in an informal or formal way and also written reports on a regular basis are mandatory. The topic will be discussed with the supervisor and, ideally, it is already into the direction of a future Master's thesis.

Course Prerequisites

Basic knowledge from a mathematical Bachelor's degree, in particular from the modules Algebra, Analysis 1-2, Linear Algebra, Numerical Analysis 1, Numerical Analysis 2, and also programming skills can be beneficial.

Times and Formalities

Upon consultation.

Reading Course Statistics/Stochastics

Course Code: 03-M-RC-STS

Prof. Dr. Werner Brannath & Prof. Dr. Thorsten Dickhaus

Contact: brannath@uni-bremen.de

Course Description

The aim of the reading course is to introduce the students to specific topics that may be relevant for the Master's thesis, using mainly original English-language literature (scientific articles and reference books). The participants are expected to work independently (with the advice of their supervisors) on the topic, give a lecture on it and prepare an term paper. Prof. Brannath and Prof. Dickhaus will announce topics for lectures in the first meeting and Stud.IP.

If you are interested in topics of other lecturers (e.g. Marc Keßeböhmer, Vanessa Didelez, Iris Pigeot or Marvin Wright), please contact these lecturers directly, ideally before our first meeting. Students can also consider their own suggestions for topics, but they must also discuss these with one of the lecturers from Stochastics or Statistics (as potential Master's thesis supervisors) best before the first meeting. The assignment of the topics as well as the scheduling of all lectures will take place in the first meeting.

Course Prerequisites

Subject-related prerequisites are basic knowledge of stochastics and statistics, as taught e.g. in the lectures "Stochastics" and "Basics of Mathematical Statistics" at the University of Bremen.

Times and Formalities

The Reading Course Statistics/Stochastics will take place in the form of a seminar. Dates and topics will be determined in a preliminary meeting at the beginning of the semester. The day and time of this meeting will be announced via Stud.IP, latest a week before the semester starts. Registration at Stud.IP is therefore a necessary prerequisite for participation.

List of Literature

The literature will be announced via Stud.IP, latest with the first meeting.

Introduction to R

Course Code: 03-M-GS-7

Prof. Dr. Werner Brannath, Eike Voß

Contact: brannath@uni-bremen.de, evoss@uni-bremen.de

Course Description

The course focuses on the basics of the statistical open source program R, including its core functions and syntax, so that students can gain a comprehensive understanding of the language. It is designed for students who have a fundamental understanding of programming and a basic understanding of statistics. No prior experience with R is required, making this course a great starting point for those looking to learn an open source and powerful statistical programming language. Students will learn how to conduct descriptive and exploratory data analyses by engaging in hands-on activities and practice working with real-world data sets. This practical approach helps students see the real-world applications of R and provides a solid foundation for further study in data analysis and programming.

Course Prerequisites

Fundamental understanding of programming and basic knowledge in statistics.

Times and Formalities

Tuesday 13:00-15:00, Wednesday 16:00-17:00.

List of Literature

- Introductory Statistics with R, P. Dalgaard, 2008
- R for Data Science, H. Wickham, 2017

Statistical Consulting

Course Code: 03-M-GS-5

Dr. Martin Scharpenberg
Contact: mscharpenberg@uni-bremen.de

Course Description

In this seminar we invite researchers from Bremen who are looking for statistical advice. In the seminar sessions we then discuss the studies/investigations of the researchers and give advice on aspects of study design, implementation, evaluation and interpretation. Students thus have the opportunity to learn about and help work through real-world problems.

Times

Freitag, 10 - 12 Uhr